Comment on 'Coulomb torque—a general theory for electrostatic forces in many-body systems'

2004 J. Phys. A: Math. Gen. 37 6409

(http://iopscience.iop.org/0305-4470/37/24/N02)

J. Phys. A: Math. Gen. **37** (2004) 6409–6411 PII: S0305-4470(04)69346-2

COMMENT

Comment on 'Coulomb torque—a general theory for electrostatic forces in many-body systems'

Michael A Jensen and Karl F Warnick

Department of Electrical and Computer Engineering, 459 Clyde Building,
Brigham Young University, Provo, UT 84602, USA

Received 23 September 2003
Published 2 June 2004
Online at stacks.iop.org/JPhysA/37/6409
doi:10.1088/0305-4470/37/24/N02

Abstract

We point out an error and several inconsistencies in the analysis of Khachatourian and Wistrom. In their force computation, an essential contribution was neglected, leading to an erroneous nonzero torque prediction. Furthermore, the analysis includes several internal contradictions.

PACS numbers: 41.20.Cv, 03.50.De, 45.50.Jf

We have read with great interest the recent paper by Khachatourian and Wistrom [1], and were particularly intrigued by the conclusion on p 6508 that 'if the direction of the electric field is taken to be normal to the equipotential surface this introduces a restrictive assumption *a priori* that does not permit electrostatic rotation contrary to experimental observations and theoretical analysis presented here'. This statement, motivated by the mathematical analysis in section 2.4 on Coulomb torque for three charged spheres, is clearly significant. However, we disagree with both the analysis and the conclusion, and therefore would like to point out some inconsistencies in the assumptions and the analysis provided in this paper.

First, we note what appears to be an important omission in (28) of [1]. The integrand with respect to Q_1 in this expression is the component of the force on each differential element of charge on sphere 1 that is orthogonal to the vector $\vec{a}_1$. In actuality, the total force on the differential element of charge is the sum of the forces created by charges on spheres 2 and 3 as well as the *force created by other charges on sphere 1*. This latter component is missing in (28) of [1]. The corrected equation should read

$$\vec{T}_1 = K \int \mathrm{d}Q_1 \vec{a}_1 \times \left[\int \mathrm{d}Q_1 \frac{\vec{R}_{11}}{R_{11}^3} + \int \mathrm{d}Q_2 \frac{\vec{R}_{12}}{R_{12}^3} + \int \mathrm{d}Q_3 \frac{\vec{R}_{13}}{R_{13}^3} \right]. \qquad (1)$$

Note that in equation (15) of [1], this self-force need not be included.

To see that equation (1) must be zero, let $\vec{r}_1$ be the integration point in the outermost integral of (1). Applying the gradient operator with respect to $\vec{r}_1$ to the expression for V_1 in equation (1) of [1] leads to

$$\nabla_1 V_1 = \int \mathrm{d}Q_1 \nabla_1 \frac{1}{R_{11}} + \int \mathrm{d}Q_2 \nabla_1 \frac{1}{R_{12}} + \int \mathrm{d}Q_3 \nabla_1 \frac{1}{R_{13}}$$

$$= \int \mathrm{d}Q_1 \frac{\vec{R}_{11}}{R_{11}^3} + \int \mathrm{d}Q_2 \frac{\vec{R}_{12}}{R_{12}^3} + \int \mathrm{d}Q_3 \frac{\vec{R}_{13}}{R_{13}^3}.$$

We see that the quantity in square brackets in (1) above is equal to $\nabla_1 V_1$. Because the potential V_1 is constant on the surface of sphere (1), the gradient must be normal to the surface. We therefore obtain $\hat{a}_1 \times \nabla_1 V_1 = 0$ and conclude that the integrand of the outer integral over Q_1 of (1) vanishes at every point $\vec{r}_1$ on the surface of sphere 1. As the surface of sphere 1 is the domain of integration, the integral must vanish, and we have $\vec{T}_1 = 0$.

We can reach the same conclusion using simple physical arguments. The quantity in square brackets in (1) is the force on a unit point charge located at a point $\vec{r}_1$ on sphere 1 due to all other charges on spheres 1, 2 and 3. Suppose that this force had a nonzero component tangential to the surface of sphere 1. Then the charge located at $\vec{r}_1$ would move in the direction of this tangential component. However, this contradicts the results obtained in sections 2.1 and 2.2 of [1], which uses the equipotential property of the sphere surfaces to construct the *static* (and therefore motionless) charge distribution. Thus, the vector quantity in square brackets in (1) must be normal to the surface of sphere 1 at the point $\vec{r}_1$. Because the radial vector $\vec{a}_1$ is also normal to the surface of sphere 1 at $\vec{r}_1$, the cross product of the vector $\vec{a}_1$ with the quantity in square brackets must vanish. We again find that the integrand of the outer integral over Q_1 vanishes at every point on sphere 1. The conclusion is that a tangential force (electric field) cannot exist on charges occupying an equipotential surface, which is a fundamental aspect of the classical electrostatic theory.

In addition to this mathematical oversight, there are several logical inconsistencies presented in the paper that warrant discussion:

1. In the proposed scenario, once the spheres are charged, no additional current flows in the system, implying that no power flow occurs. However, if a torque were to exist on the spheres and they were allowed to spin, this implies that work is being performed for no energy input. This violates conservation of energy.
2. If indeed a torque were to exist on the surface charge on a sphere, this does not imply that the actual conducting sphere would rotate. Rather, since the charge moves freely within the conductor, the effect would be a *current flowing on the conductor surface*, and not conductor motion. Perhaps a finite conductivity would lead to some conductor motion, but the source of the rotation would then require the existence of a current on the sphere and the dissipation of energy as well.
3. The entire derivation uses electrostatic principles. However, moving charges represent a dynamic system. If the conclusion of this paper were correct, then the volumetric charge distribution would have to evolve in time. However, this contradicts the concept of an equilibrium charge distribution as computed in sections 2.1 and 2.2. It is inconsistent to use a static charge distribution to predict a dynamic one.

Given these errors and contradictions, one must question the experimental observation of sphere rotation reported by the authors. Certainly, there would be repulsive/attractive forces between the spheres, and perhaps the resulting pendulum motion of the spheres suspended by the wires led to rotation due to slight bends in the metal wires or asymmetric distribution of the sphere weight about the point of attachment to the wire. Other explanations could certainly be identified. Regardless, the incorrect conclusions offered in the present paper do not explain the experimental observation reported.

References

[1] Khachatourian A V M and Wistrom A O 2003 Coulomb torque—a general theory for electrostatic forces in many-body system *J. Phys. A: Math. Gen.* **36** 6495–508